

### Math 43 Midterm 3 Review

[1] [a] Find  $a_{11}$  for the arithmetic sequence with  $a_3 = x - 2y$  and  $a_8 = 4y - 3x$ .

[b] Find  $a_{13}$  for the geometric sequence with  $a_5 = \frac{x^8}{2y^5}$  and  $a_8 = 4x^3y$ .

[2] Find the sum of the series  $-73 - 66 - 59 - 52 \cdots + 529$ .

[3] Find the value of  $\sum_{n=3}^8 (-1)^n n(n-4)$ .

[4] Prove by mathematical induction:

[a]  $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

for all integers  $n \geq 1$

[b]  $\sum_{i=0}^n (2i+1)3^{i-1} = \frac{1+n3^{n+1}}{3}$

for all integers  $n \geq 0$

[c]  $a + ar + ar^2 + \cdots + ar^n = \frac{a(r^{n+1} - 1)}{r - 1}$

for all integers  $n \geq 1$

[d]  $\sum_{i=1}^n \frac{3}{(i+3)(i+2)} = \frac{n}{n+3}$

for all integers  $n \geq 1$

[5] Write the repeating decimal  $0.4\overline{72}$  as a simplified fraction. **NOTE: Only the 72 is repeated.**

[6] Completely simplify [a]  $\binom{200}{4}$  [b]  $\binom{2n+2}{3}$ .

[7] Use sigma notation to write the series  $\frac{1}{7 \cdot 3} + \frac{1}{4 \cdot 6} + \frac{1}{1 \cdot 12} - \frac{1}{2 \cdot 24} - \cdots - \frac{1}{17 \cdot 768}$ .

[8] Find the coefficient of  $x^{34}$  in the expansion of  $(2x^5 - 3x^2)^{11}$ .

[9] Find the value of  $\sum_{n=3}^{\infty} 4(0.97)^{2n-1}$ . **HINT: Write out the first few terms first.**

[10] Find the first 5 terms of the sequence defined recursively by  $a_n = 2a_{n-1} - 3$ ,  $a_1 = 4$ .  
Is the sequence arithmetic, geometric or neither? Explain how you arrived at your conclusion.

[11] Use Pascal's triangle and the Binomial Theorem to expand and simplify

[a]  $(3x - 2y)^6$

[b]  $\left(\sqrt{x} - \frac{2}{x}\right)^4$

[12] EJ bought a new car in 1998. The registration fee was \$800 that year. Each year, the registration fee decreased by 10%. The car was eventually sold for scrap in the year when its registration fees were \$3.34. What year was EJ's car sold for scrap?

[13] CJ and DJ both just graduated from college and started new jobs. Neither could afford the market rate for apartment rentals, so they worked out deals with their landlords. CJ agreed to pay \$400 rent the first month, and each month after, \$7 more rent than the previous month. DJ agreed to pay \$380 rent the first month, and each month after, 2% more rent than the previous month. After 2 years, who will have paid more rent altogether, and by how much?

## Answers

[1]      [a]       $\frac{38}{5}y - \frac{27}{5}x$       [b]       $\frac{128y^{11}}{x^{\frac{16}{3}}}$

[2]      19,836

[3]      21

[4]      PROOFS INCLUDED IN UPCOMING SOLUTIONS HANDOUT

[5]       $\frac{26}{55}$

[6]      [a]      64,684,950      [b]       $\frac{4}{3}n^3 + 2n^2 + \frac{2}{3}n$

[7]       $\sum_{n=1}^9 \frac{1}{3(10-3n)(2)^{n-1}}$

[8]      -11,547,360

[9]      58.1207

[10]      4, 5, 7, 11, 19  
neither arithmetic nor geometric

[11]      [a]       $729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6$

[b]       $x^2 - 8x^{\frac{1}{2}} + 24x^{-1} - 32x^{-\frac{5}{2}} + 16x^{-4}$

[12]      2050

[13]      DJ will have paid \$28.31 more rent